Semi-global Leaderless Consensus with Input Saturation Constraints via Adaptive Protocols

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ABSTRACT: *In this paper, the problems of semi-global leaderless consensus for continuous-time multi-agent systems under input saturation restraints are investigated. We consider the leaderless consensus problems under fixed undirected communication topology. New necessary synthesis conditions are established for achieving semi-global leaderless consensus via the distributed adaptive protocols, which is designed by utilizing low gain feedback tactics and the relative state measurements of the neighboring agents. Finally, simulation results illustrate the theoretic developments.*
Keywords -*Semi-global consensus; Multi-agent systems; Inputsaturation; Adaptive protocols*

I. INTRODUCTION

During the last few decades, consensus has been became a very hot research topic inmany scientific fields, such as computer science, physics, biology,and system control. Roughly speaking, consensus means the agreementof agents on the joint status value via utilizing local interactionfrom the neighbouring agents. In recent years, extensive research attentionhas been paid into the consensus problem of multi-agent systems[1-5]. It should be noted that, although thepower of each individual agent is quite limited, the wholeinteraction network system may accomplish complex tasks in acoordinated fashion. Therefore, the consensus problem has a widerange of potential applications, to mention just a few, includingcooperation of unmanned air vehicles (UAVs) in [6],distributed sensor networks in [7] and [8],attitude alignment for a flock of satellites in [9],flocking control in [10] and [11], congestion control under communication topology [13].Interested readers are referred to the recent survey articles [14], [15] and the references citedtherein for details.It is known that consensus has been studied for several decades in computer science. Broadlyspeaking, there are two classes of consensus, that is, the leaderless consensus and the leaderfollowing one. Usually, the multi-agent system dynamic models are described by double-integratordynamics or single-integrator kinematics [13]-[17]. In recent literature, the general linear dynamicmodel has been considered to address more complicated system dynamics in engineering practice.For instance, in [18] the authors studied the consensus problems of both leaderless and leader-following cases in the situation of fixed interaction networks. In [19] the consensus problemsin the case of leader-following were discussed under a switching communication network. Theleaderless nonlinear consensus subject to parametric uncertainties was achieved by developingadaptive continuous finite-time distributed protocols in [20]. The consensus of multi agents withsecond-order, linear and Lipschitz nonlinear dynamics via utilizing the adaptive algorithms werestudied in [21] and [22], respectively. A distributed observer based algorithm was proposed in [23]for leader-following consensus of networked multiple agents systems.

Indeed, up to now, the investigation on consensus has many comprehensive achievements.However, in practice networked multi-agent systems may have more complicated dynamics. Theindustrial applications of consensus are limited by all kinds of physical conditions, such as the inputsaturation constraints. Till now, more and more research activities has paid close attention tothe consensus problems for networked multiple agents systems under input saturation constraintsbecause of its unique characteristic and practical applications. In [24], the authors investigated theglobal consensus of linear networked multiple agents dynamics with input saturation by distributedobserver-based algorithm over fixed and switching interaction networks. For discrete-time systems,the global consensus problem under input saturation constraints was addressed in [25]. In [26] Suet al. proposed a linear low gain feedback algorithm to reach semi-global consensus for networkedmulti agents dynamics under the assumption of agents are linear asymptotically null controllablewith bounded controls (ANCBC) [27]. While in recent paper [28], the semi-global consensus oflinear dynamics in the case of input saturation was studied by utilizing observer-based algorithmsand the low gain feedback technique, under the assumptions that the communication networkswere jointly connected or connected and each agent was detectable. With the input saturationrestricts, the semi-global synchronization problem for multiple agents with linear systems wasstudied in [29].

In the present paper, we revisit the problem of leaderless consensus of linear multi agentsdynamics with input saturation constraints. New semi-global consensus conditions are establishedby using distributed adaptive

protocols. Compared with some existing results, the distinctivepoints of the paper can be expressed as the following sentences. Under the assumption thatthe agents are ANCBC, without realizing any information of the networks, all agents achievean agreement by utilizing the distributed adaptive protocols and using the low gain feedbacktechnique over both fixed and switching interaction networks. Also, we provide simulation resultson two numerical examples to verify the theoretical developments.

The remainder of this paper is arranged as follows. Preliminaries and notations are introducedin Section 2. In Section 3, based on the distributed adaptive protocols, we address the problemof semi-global leaderless consensus under fixed undirected graph.The simulation results on two numericalexamples are given in Section 4 to verify the theoretic developments. Finally, a brief conclusionis drawn in Section 5.

II. PROBLEM BACKGROUND AND PRELIMINARIES

In this paper, we suppose that the information exchange between agents and the leader is described by a fixed undirected graph $G = (V, E, A)$, with the set of vertices $V = \{1, 2, ..., N\}$ whose elements represent agents of the group, the set of undirected edges $E \subset V \times V$ represent neighboring relations among agents, and a weighted adjacency matrix $A = [a_{ij}] \in R^{n \times n}$ is defined as $a_{ij} > 0$ if and only if $(j, i) \in E$; otherwise, $a_{ij} = 0$. Here, we also assume that $a_{ij} = a_{ji}$ for all $i = 1, 2, ..., n$.

For an undirected graph G, a Laplacian matrix $L = [l_{ij}] \in R^{n \times n}$ of graph G is defined as $L = D - A$, with the degree matrix $D = diag\{d_1, \ldots, d_N\}$ is defined as i th diagonal elements $\sum_{j=1, j \neq i}^{N} a_{ij}(t)$. Define $B_0 = diag(b_{10}(t), b_{20}(t), ..., b_{N0}(t))$ as $b_{i0} > 0$ if the leader is available by agent i and $b_{i0} = 0$ otherwise. The eigenvalues of L can be ordered as $\eta_1 \leq \eta_2 \leq \cdots \leq \eta_N$. Then, $\eta_1 = 0$ with its corresponding right eigenvector $1 = [1, 1, ..., 1]^T \in R^n$. Moreover, $\eta_2 > 0$ if G is a connected graph.

Assume that M is a matrix, the notation M^T denotes its transpose matrix. A symmetric positive matrix M indicates that all its eigenvalues are positive, a positive semi-definite M indicates that all its eigenvalues are non-negative. The symbol $\otimes$ is the Kronecker product. I_n denotes the identity matrix of dimension $N \times N$.

Lemma 1 [4]:Denote L as the Laplacian of a fixed undirected graph G include N agents, denote $\bar{L}$ as the Laplacian of the graph consisting of N agents and one leader, $b_{i0} > 0$ if there is a spanning tree with the leader as the root vertex.

Lemma 2 [26]: For any $\psi \in R^{m \times m}$ and $\zeta_i, \ell_i \in R^m, i = 1, 2, ..., N$

$$\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(t)(\zeta_i - \zeta_j)^T (\psi(\varepsilon)(\ell_i - \ell_j)) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(t) \zeta_i^T (\psi(\varepsilon)(\ell_i - \ell_j))$$

In this paper, we consider an MAS of N identical continuous-time agents subject to input saturation constraints, moving in an n-dimensional Euclidean space. The dynamic of each agent is modeled by

$$\dot{x}_i = Ax_i + B\sigma_\Delta(u_i)$$
$$y_i = Cx_i, \quad i = 1, 2, ..., N \tag{2}$$

where $x_i \in R^n$, $y_i \in R^P$ are the state of agent i and the measurement output of agent i respectively, $u_i \in R^m$ is the control input, $\sigma_\Delta : R^m \to R^m$ is a saturation function defined as $\sigma_\Delta(u_i) = sgn(u_{ij}) \min\{|u_{ij}|, \varpi\}$ with a scalar value $\Delta > 0$. For $u_i = (u_1, u_2, ..., u_m)^T$, $\sigma_\Delta(u_i) = (\sigma_\Delta(u_1), \sigma_\Delta(u_2), ..., \sigma_\Delta(u_m))^T$.

To address the leaderless consensus, the average state of N agents is defined as follows

$$\dot{\bar{x}}_i = \frac{1}{N} \sum_{i=1}^{N} x_i, i = 1, 2, ..., N \tag{3}$$

Definition 1. To address the semi-globalleaderless consensus problem, it means that, for any prior given compact set $\chi \in R^n$, we can design a distributed control law uiby only using the partial information of agent i from neighbors, such that all the agents can reach the same state asymptotically, i.e.,

$$\lim_{t \to \infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad i = 1, 2, \ldots, N$$

with any initial condition $x_i(0) \in \chi, i = 0, 1, 2, \ldots, N$

In order to employ the low gain feedback technique, we need to suppose that the pair (A,B)satisfies the following Assumption 1. For more details, the reader can refer to the book [27].

Assumption 1: The pair (A, B) is ANCBC and each eigenvalue of the matrix A is on theimaginary axis simple.

Lemma 3[29]: Under Assumption 2. A unique solution $P(\varepsilon)$ is derived by the following parametric ARE.

$$A^T P(\varepsilon) + P(\varepsilon)A - \gamma P(\varepsilon)BB^T P(\varepsilon) = -\varepsilon P(\varepsilon) \tag{4}$$

where $\gamma > 0$ is a positive scalar. Furthermore, we have $\lim_{\varepsilon \to 0} P(\varepsilon) = 0$.

Lemma 4[29]: Let Assumption 1 hold, for any $\gamma > 0$ and $\varepsilon \in (0, 1]$, the unique solution in Lemma 3 is $P(\varepsilon) = W^{-1}(\varepsilon)$ which satisfies a Lyapunov function

$$W(A + \frac{\varepsilon}{2}I_n)^T + (A + \frac{\varepsilon}{2}I_n)W = \gamma BB^T \tag{5}$$

Assumption 2: The graph G is connected at all time.

III. MAIN RESULTS

This section investigates the semi-global leaderless consensus with input saturation under a fixedinteraction network topology. The adaptive control algorithm will be employed to address the problem.Now, motivated by the distributed consensus protocol with an adaptive control law in [22], we are ready to design a distributed consensus algorithm as follows:

$$u_i = K \sum_{j=1}^{N} a_{ij} m_{ij} \left(x_i - x_j \right) \quad \dot{m}_{ij} = \beta_{ij} a_{ij} \left(x_i - x_j \right)^T \hat{K} \left(x_i - x_j \right) \tag{6}$$

where $\beta_{ij} = \beta_{ji}$ are the plus quantities, m_{ij} is the adaptive coupling weights, the feedbackmatrix K and $\hat{K}$ are designed as $K = -B^T P(\varepsilon)$ and $\hat{K} = -P(\varepsilon)BB^T P(\varepsilon)$ respectively, $P(\varepsilon)$ is theunique solution that can be obtained from the algebra Riccati equation (4).

By introducing the above contents, now we propose the results as follows.

Theorem1. Suppose that Assumptions 1 and 2 are satisfied. Then the multi-agent system(2) of N agents achieve semi-global leaderless consensus under the distributed control law (6).

Proof: Let us define $\delta_i = \left[\delta_1^T, \delta_2^T, \ldots, \delta_N^T \right]^T$ with $\delta_i = x_i - \overline{x}_i$. Then, we have the following equations

$$\dot{\delta}_i = \dot{x}_i - \dot{\overline{x}}_i = A\delta_i + B\sigma_\Delta(u_i) - \frac{1}{N} \sum_{i=1}^{N} B\sigma_\Delta(u_i)$$

$$\dot{m}_{ij} = \beta_{ij} a_{ij} \left(\delta_i - \delta_j \right)^T \hat{K} \left(\delta_i - \delta_j \right) \tag{7}$$

Now, for the multi-agent system (2), let us consider a common Lyapunov function candidate

$$V = \sum_{i=1}^{N} \delta_i^T P(\varepsilon) \delta_i + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{\left(m_{ij} - \omega \right)^2}{2\beta_{ij}} \tag{8}$$

The matrix $P(\varepsilon)$ is the unique solution from the parametric algebra Riccati equation (6), ω is a positive parameter to be designed. By way of convenient notation, define c be a positive number such that

$$c \geq \sup_{\varepsilon \in (0,1], x_i(0) \in \chi, i=1,2,...,N} \sum_{i=1}^{N} \left\{ \delta_i^T P(\varepsilon) \delta + \sum_{j=1, j \neq i}^{N} \frac{(m_{ij} - \omega)^2}{2\beta_{ij}} \right\}$$

(9)

Such a c exists since χ is bounded and $\lim_{\varepsilon \to 0} P(\varepsilon) = 0$. Let

$$\varsigma_c := \left\{ \delta \in R^{Nn} : V(\delta) \leq c \right\}$$

(10)

and $\varepsilon^* \in (0,1]$ be such that, for each $\varepsilon \in (0, \varepsilon^*]$, $e \in \varsigma_c$ implies that

$$\left\| B^{\mathrm{T}} P(\varepsilon) \sum_{j=1}^{N} m_{ij} a_{ij} (x_i - x_j) \right\|_{\infty} \leq \Delta, \ i = 1, 2, ..., N$$

(11)

Therefore, by Lemma 3, consider any $\varepsilon \in (0, \varepsilon^*]$, for any $e \in \varsigma_c$, the derivative of V can be evaluated as

$$\dot{V} = \sum_{i=1}^{N} \dot{\delta}_i^{\mathrm{T}} P(\varepsilon) \delta_i + \sum_{i=1}^{N} \delta_i^{\mathrm{T}} P(\varepsilon) \dot{\delta}_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{(m_{ij} - \omega)}{\beta_{ij}} \dot{m}_{ij}$$

$$= \sum_{i=1}^{N} \delta_i^{\mathrm{T}} (P(\varepsilon) A + A^{\mathrm{T}} P(\varepsilon)) \delta_i - 2 \sum_{i=1}^{N} \delta_i^T P(\varepsilon) BB^T P(\varepsilon) \sum_{j=1}^{N} m_{ij} a_{ij} (\delta_i - \delta_j)$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{(m_{ij} - \omega)}{\beta_{ij}} \beta_{ij} a_{ij} (\delta_i - \delta_j)^T P(\varepsilon) BB^T P(\varepsilon) (\delta_i - \delta_j)$$

(12)

By Lemma 2, for any $\zeta_i \in R^{\mathrm{m}}, \ell_i \in R^{\mathrm{m}}, i = 1, 2, ..., N$, we have

$$\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(t) (\zeta_i - \zeta_j)^T (\ell_i - \ell_j) = \xi^{\mathrm{T}} (L(t) \otimes I_m) \ell$$

where $\xi = \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \end{bmatrix}^{\mathrm{T}}$ and $\ell = \begin{bmatrix} \ell_1 & \ell_2 & \cdots & \ell_N \end{bmatrix}^{\mathrm{T}}$. Therefore, we can continue (12) as follows:

$$\dot{V} = \sum_{i=1}^{N} \dot{\delta}_i^{\mathrm{T}} P(\varepsilon) \delta_i + \sum_{i=1}^{N} \delta_i^{\mathrm{T}} P(\varepsilon) \dot{\delta}_i - 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \omega a_{ij} \delta_i^T P(\varepsilon) BB^T P(\varepsilon) (\delta_i - \delta_j)$$

$$= \delta^T (I_N \otimes (P(\varepsilon) A + A^{\mathrm{T}} P(\varepsilon) - 2\omega L \otimes P(\varepsilon) BB^{\mathrm{T}} P(\varepsilon)) \delta$$

Since the graph is connected, zero is a simple eigenvalue of L and all the other eigenvalues are positive. Let $U \in R^{N \times N}$ be such a unitary matrix that $U^T L U = \Lambda =: diag(0, \eta_2, ..., \eta_N)$. Because the right and left eigenvectors of L corresponding to the zero eigenvalue are 1 and 1^T, respectively, we can choose

$$U = \begin{bmatrix} \frac{1}{\sqrt{N}} & Y_1 \end{bmatrix} \quad \text{and} \quad U^T = \begin{bmatrix} \frac{1^T}{\sqrt{N}} \\ Y_2 \end{bmatrix}, \quad \text{with} \quad Y_1 \in R^{N \times (N-1)} \quad \text{and} \quad Y_2 \in R^{(N-1) \times N}. \quad \text{Let}$$

$\xi = \begin{bmatrix} \xi_1^T, ..., \xi_N^T \end{bmatrix}^T = (U^T \otimes I_n) \delta$. Then we have

$$\dot{V} = \delta^T (I_N \otimes (P(\varepsilon) A + A^{\mathrm{T}} P(\varepsilon) - 2\omega L \otimes P(\varepsilon) BB^{\mathrm{T}} P(\varepsilon)) \delta$$

$$\leq \sum_{i=2}^{N} \xi_i^T \left(P(\varepsilon) A + A^{\mathrm{T}} P(\varepsilon) - 2\omega \eta_i P(\varepsilon) BB^{\mathrm{T}} P(\varepsilon) \right) \xi_i$$

(13)

By Lemma 3 choosing $\gamma \leq 2\omega \eta_i, i = 2, ..., N$, (13) can be continued as :

$$\dot{V} \leq \sum_{i=2}^{N} \xi_i^T \left(P(\varepsilon) A + A^{\mathrm{T}} P(\varepsilon) - 2\gamma P(\varepsilon) BB^{\mathrm{T}} P(\varepsilon) \right) \xi_i = -\varepsilon \sum_{i=1}^{N} \delta_i^T P(\varepsilon) \delta_i < 0$$

Thus, semi-global consensus is achieved, i.e., such that for a set of original condition $x_i(0) \in \chi$

$$\lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0, \quad i = 1, 2, ..., N$$

This completes the proof.

IV. SIMULAION STUDY

Herein the simulation experiments are supplied to clarify the theoretic results on the leaderlessconsensus developed in Section 3. Consider a multi agents system consisting of eight agents.Assume that the dynamic of every agent holds the form of (2) with the following parameters:

$$A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Obviously the pair (A, B) satisfies Assumption 1. Suppose the topology is considered as in Fig.1. The initial states of eight agents are randomly assigned. Thus, We chose $\beta_{ij} = 1$ and $\omega = 5$, $\eta_2 = 0.2907$ can be obtained by using an exhaustive method. Then $\gamma \leq 2\omega\eta_2 = 2.907$. For the situation of $\varepsilon = 0.2$, then we get a set of solutions as

$$P(0.2) = \begin{bmatrix} 0.3429 & -0.3086 \\ -0.3086 & 0.5556 \end{bmatrix}$$

Thus, the feedback gain K and $\hat{K}$ as

$$K = \begin{bmatrix} 0.3086 & -0.5556 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} 0.0952 & -0.1715 \\ -0.1715 & 0.3087 \end{bmatrix}$$
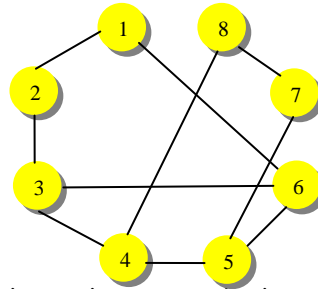

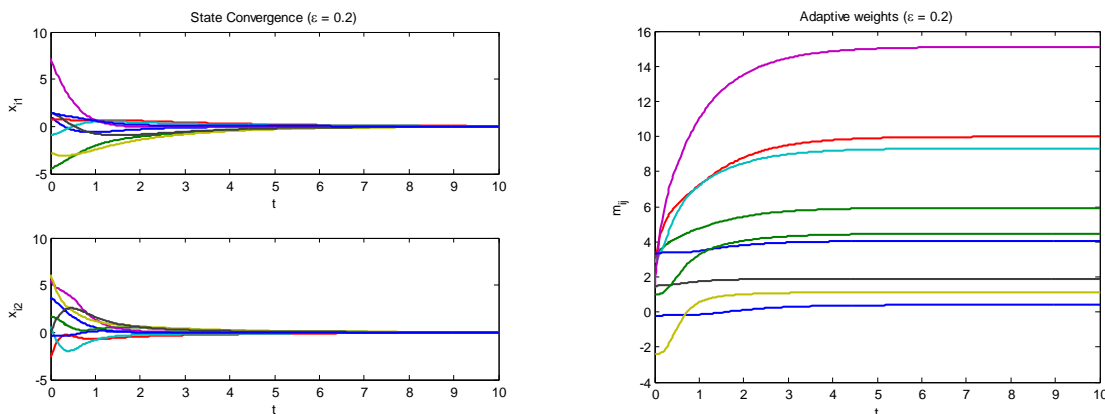Fig. 1: The interaction communication network.

Fig. 2: The states of eight agents.Fig. 3: The adaptive weights m_{ij}.

Under the control law (6), the trajectories of the state difference between agents are shown in Fig. 2. It is obvious that the semi-global leaderless consensus can be achieved.

For the situation of $\varepsilon = 0.2$, we can get the unique solution

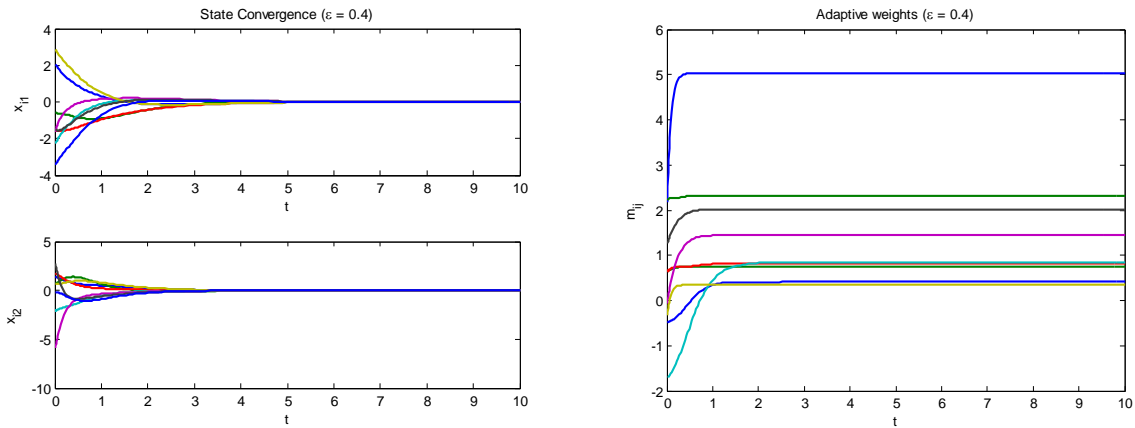$$P(0.4) = \begin{bmatrix} 0.4883 & -0.3906 \\ -0.3906 & 0.6250 \end{bmatrix}$$

Fig. 2: The states of eight agents. Fig. 3: The adaptive weights m_{ij}.

Thus, the feedback gain K and $\hat{K}$ as

$$K = \begin{bmatrix} 0.3906 & -0.6250 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} 0.1526 & -0.2441 \\ -0.2441 & 0.3906 \end{bmatrix}$$

Under the control law (6), it is obvious that the semi-global leaderless consensus can be achieved. The simulation results are in Fig. 4. The adaptive weights m_{ij} under $\varepsilon = 0.2$ and $\varepsilon = 0.4$ are shown in Fig. 3 and Fig. 5 respectively.

V. CONCLUSION

In this paper, we consider the semi-global leaderless consensus problem of ANCBC multipleagents system subject to input saturation restraints both over fixed and switching topology. Theadaptive control law is proposed by utilizing a low gain feedback strategy. The connectivity of thegraph in fixed networks and the joint connectivity of the graph in switching networks are the keyrequirements to guarantee the semi-global leaderless consensus. Finally, the theoretical resultswere verified by simulation study.

VI. Acknowledgements

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